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Abstract:

Linear models have produced equivocal results on the mean-reversion hypothesis in Zambia casting doubt on the practical implications of the purchasing power parity theory. This study revisits the mean-reversion hypothesis for the real effective exchange rate (REER) using non-linear models over the floating exchange rate period (2000-2020). A battery of non-linearity tests confirms the existence of non-linear dynamics and nonlinear unit root tests also establish that the REER is stationary and therefore, mean reverting. The exponential smooth transition autoregressive model is selected and estimates threshold values from which to judge whether REER deviations are large enough to induce arbitrage and therefore, adjustment back to equilibrium. The study recommends that nonlinear dynamics should be part of the REER analysis.

JEL Classification: F310, G15, F41
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Contents

1. Introduction

Real exchange rates are important to policy makers as they are used to assess economic competitiveness, exchange rate misalignments and for setting parities among economies considering integration into a monetary union (Rogoff, 1996). The dynamics of the RER are underpinned by the theory of purchasing power parity (PPP), a theory widely assumed to hold in open economy macroeconomic models and is defined in its absolute or relative form (Papell, 2002). The absolute version states that "the equilibrium exchange rate between domestic and foreign currencies equals the ratio between domestic and foreign prices" while the relative version relates " equilibrium changes in the exchange rate to changes in the ratio of domestic and foreign prices" (Frenkel, 1978; Salvatore, 2016). Both variants of PPP conjecture that the RER reverts to a constant mean (mean reversion[2]). The appropriate model for analysing RER dynamics for researchers and policy makers alike is challenging because there is insufficient empirical support for the PPP in the literature and where it exists, is fragile (Frankel & Rose, 1996; Taylor, 2002; Chortareas & Kapetanios, 2009).

Empirically, evidence of PPP or mean reversion in RER is established through linear unit root testing of the RER series or cointegration testing methodologies[3]. However, studies using these methodologies but focusing on the floating exchange rate period (post-1973 in most advanced economies) largely fail to find evidence of PPP as the hypotheses tests lack power. Lack of power in unit root tests and cointegration tests may arise due to relatively short sample spans, or the incorrect application of linear models on a series whose true data generating process is nonlinear (Sarno, 2005). As such, increasing sample size in linear models or application of nonlinear models are the two notable ways researchers tried to solve the low power problem. Evidence from these two tested solutions are discussed next.

The short sample span problem was resolved by researchers' recourse to long-span studies (at least 70 years of data) and panel studies (pooling several country currencies in the floating exchange rate period), albeit with wavering success. While long-span studies using cointegration tests find more evidence of mean reversion (Frankel, 1990; Diebold et al. 1991; Cheung & Lai, 1994; Lothian & Taylor, 1996; Taylor, 2002), regime shifts and structural breaks by mixing periods of fixed and floating exchange rates may invalidate inference and conclusions (Froot & Rogoff, 1994; Sarno & Taylor, 2002). In comparison, unit root tests in panel studies obviate the critique of regime shifts but have produced mixed evidence on PPP (Abuaf & Johnson, 1990; Frankel & Rose, 1996; Taylor & Sarno, 1998; O'connell 1998; Papell & Theodoridis, 2001; Papell, 2002). Even when panel studies find evidence in support of PPP, they are inconclusive at best because rejection of the null of joint non-stationarity in a panel unit root test does not imply that all series are jointly stationary as alternative hypotheses require at least one series be stationary (Taylor & Sarno, 1998)[4]. Taylor & Sarno (1998) demonstrate that it is quite common for only one stationary series in a panel to drive the

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[2] Mean reversion exists when PPP holds. These two terms are synonymously used in this study

[3] Unit root tests are effectively a test for whether the RER is mean reverting (stationary) while cointegration tests are tests of long-run equilibrium among the components that make up the RER.

[4] Chortereas & Kapetanios (2009) devise a way in which to identify stationary and non-stationary series within a panel while maintaining the benefits of using a panel and is the first study to conclusively find evidence of PPP in a panel framework.

panel unit root test towards rejection of the null. Given the above, linear models even with long sample spans have not successfully resolved the question of whether PPP holds.

In contrast to linear models, the use of nonlinear models, and in particular nonlinear unit root tests, has shown relatively more success in finding evidence of PPP in short sample spans (floating exchange rate period only) without loss of power. Most importantly, nonlinear modelling of the RER has been shown to produce evidence in support of PPP in settings where linear unit root tests or cointegration tests have rejected the PPP (Sarno, 2000; Taylor et al., 2002; Liew et al., 2003, 2004; Anoruo et al., 2006; Bahmani-Oskooee, 2008). One of the reasons why linear models have not been as successful in modelling PPP is that they assume away transaction costs. In Linear models, any RER deviation from equilibrium, no matter its size, will necessarily induce arbitrage opportunities which when exploited, will eventually drive the RER back to equilibrium at a constant rate. Realistically, the PPP deviations must be large enough to overcompensate the transaction costs before arbitrage opportunities can be exploited by economic agents. Transaction costs will include aspects such as transportation costs, sunk costs of international trade as well as tariff and non-tariff barriers to trade (Sercu et al., 1995). Therefore, in nonlinear models the speed of adjustment of RER to equilibrium depends on the size of the PPP deviation[5] and this flexibility gives nonlinear models an edge over standard linear models (Sarno, 2005). There are two important caveats to note when using nonlinear models. Incorrectly assuming nonlinearity and the use of wrong nonlinear specification have worse outcomes for hypothesis testing than if the nonlinearity is ignored it altogether (Enders, 2014). Therefore, before using a nonlinear model, one should formally test for nonlinearity and when confirmed, additional formal tests should be conducted to choose an appropriate nonlinear model among the many possible candidates.

For Zambia, evidence of PPP holding using linear models is not conclusive because Mungule (2004), Bahmani-Oskooee et al. (2008), Arize (2011), Chipili (2019) find evidence in favour of PPP while Mokoena et al. (2008) and Pamu (2011) do not[6]. Mungule (2004), Bahmani-Oskooee et al. (2008) Arize (2011), cover a period which included both floating and fixed exchange rate regimes in Zambia. Their conclusions may therefore be subject to the same criticism as those for long-span time-series studies in which inference may be invalid due to regime shifts. Pamu (2011) and Mokoena et al. (2008)[7] focus on the floating exchange rate period but their conclusions using linear tests may be subject to low power due to ignoring nonlinear dynamics underlying the RER. In this regard, only the study of Chipili (2019) is the one exception that finds evidence of PPP using the linear cointegration methodology in the floating exchange rate. However, it is not clear whether his findings are robust to the

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[5] For smaller deviations a low speed is expected while for large deviations, a faster speed of mean reversion is expected

[6] Others like Moono (2010) and Kanyembo & Sheefeni (2013) use bilateral real exchange rates between Zambia and South Africa with mixed findings too. Moono (2010) uses a relatively longer time series6 and finds no evidence of PPP while Kanyembo & Sheefeni (2013) find week evidence of PPP.

[7] Mokena et al (2008) do not strictly focus on the floating exchange rate period for Zambia as their sample begins in 1990 while floating exchange rates were introduced in 1994 and therefore, their conclusions may be compounded by the regime shifts as well.

additional check for PPP that requires that once cointegration is found, the cointegrating constants should be not be statistically different from unity in absolute terms (Mark, 1990).

Bahmani-Oskooee et al. (2008), Mokoena et al. (2008), Arize (2011) and Zerihun & Breitenbach (2016) are the only known attempts to the best knowledge of the author to apply nonlinear models to the test of PPP in Zambia. Bahmani-Oskooee et al. (2008), Mokoena et al. (2008) and Arize (2011) collectively report mixed results using nonlinear approaches. However, these studies did not test for nonlinearity but assumed it altogether, hence their conclusions are questionable. Zerihun & Breitenbach (2016) however, do test for nonlinearity and find evidence for nonlinear adjustment in the real effective exchange rate (REER). Their sample focusses on the floating exchange rate period unlike Bahmani-Oskooee et al. (2008) and Arize (2011), but among the plethora of nonlinear models, their study does not specify which one is best suited for analysing the Zambian REER. Further, none of the studies provide empirical estimates of the range of values (which proxy transaction costs) from which RER deviations may be judged to be large enough to induce arbitrage, and therefore, adjustment to equilibrium.

From these identified gaps in the literature, this study answers four questions that seek to deepen the understanding of the nonlinear dynamics present in the Zambian REER:

a) The prevalence of nonlinear adjustments in the REER if a battery of formal nonlinearity tests are used;
b) Among the plethora of nonlinear models, the most appropriate nonlinear model that characterizes the Zambian REER;
c) The range of values from which REER deviations can be judged to be small or large enough to warrant any adjustment to PPP equilibrium; and
d) Whether the PPP holds in a nonlinear sense using a more powerful nonlinear stationarity test not applied to the exchange rate during the floating period.

The study finds nonlinear adjustments in the real effective exchange rate, best modelled by an exponential smooth transition autoregressive (ESTAR) model, are important and cannot therefore be ignored during analyses to inform policy. The nonlinear unit root tests confirm that real exchange rate deviations from equilibrium are mean reverting to a “no-arbitrage band” of values estimated for the first time.

The rest of the paper is organised as follows: Section 2 describes the theoretical and empirical literature on nonlinear testing, estimation, and nonlinear unit root tests. Section 3 describes the methodology while Section 4 outlines and discusses the empirical results. Section 5 concludes.

2. Literature Review

Theoretically, the equation for the real exchange rate is given by

$$Q \equiv SP^{*}/P \tag{1}$$

where Q is the real exchange rate; S is the domestic price of foreign currency; P^* is the foreign price level; and P is the domestic price level; PPP implies $Q = 1$.

In logarithmic form, equation (1) is expressed as

$$q_t = s_t - p_t + p_t^* \qquad (2)$$

Therefore, $q_t \neq 0$ represents RER deviations from equilibrium or from PPP.

Tests for PPP using unit root tests investigate whether q_t is a stationary process (mean-reverting) or not. Disappointingly, there is overwhelming evidence in the literature rejecting PPP using linear unit root tests in both developing and developed economies (Adler & Lehman, 1983; Mark, 1990; Huzinga, 1987; Meese & Rogoff, 1988; Mungule, 2004; Mokoena et al. 2008; Pamu, 2011; Arize, 2011).

Linear specifications of equation (2) do not account for the role of transaction costs in that the adjustment of the RER to equilibrium is modelled as continuous and of a constant speed regardless of the size of the deviation from PPP. However, the speed of adjustment in nonlinear models depends on the size of the RER deviation from equilibrium (Sarno, 2005). The literature has identified trade barriers (tariff and non-tariff barriers), transportation costs and sunk costs of international arbitrage to be important sources of nonlinear adjustments in RER (Dumas, 1992; Sercu et al., 1995; Obstfeld & Rogoff, 2000; O'Connell & Wei, 2002). Nonlinearity in the RER may also reflect the tendency of traders to wait for sufficiently large international arbitrage opportunities to open before exploiting them (Dumas, 1992; O'Connell & Wei, 2002; Sarno, 2005). Therefore, there will be a range of values of PPP deviations (i.e. the "no arbitrage band"), which may not be not large enough to cover the costs of arbitrage, and therefore a perceived PPP disequilibrium may persist because it is not economically viable to exploit (Dumas, 1992). Linear models ignore this and assume disequilibrium of any magnitude is economically viable. Essentially, the no-arbitrage band will consist of upper and lower limit threshold values such that PPP deviations outside those threshold values neccessitate an adjustment to equilibrium.

Modelling transaction costs is usually done through regime-switching models such as the Threshold Autoregressive Models (TAR) and Smooth Transition Autoregressive Models (STAR). The empirical literature on PPP favours the use of STAR models because TAR models imply that any observed PPP deviation outside the "no arbitrage band" will revert to the *band* in an abrupt and instantaneous manner. However, Dumas (1992), Taylor et al. (2003) and Sarno (2005) show that time aggregation will tend to smooth the transition between the two regimes rather than be sudden. Since the RER is usually constructed with price indices made of different goods prices each with different sizes of international arbitrage costs, a smooth adjustment to the overall RER is more realistic (ibid).

STAR models popularised by Granger & Terasvirta (1993) are able to characterise nonlinear adjustment of the RER which takes place in each period, and the speed of adjustment varies with the extent of the deviation from parity. In particular, the Exponential Smooth Transition Autoregressive (ESTAR) and Logistic Smooth Transition Autoregressive (LSTAR) are preferred. ESTAR models are best suited for modelling mean reversion in the RER that is the same (symmetric) whether it is above or below its equilibrium value. If however it the RER adjusts to equilibrium in an asymmetric manner depending on whether it is above or below equilibrium, then the LSTAR is preferred. The choice between ESTAR or LSTAR has to be done using formal testsing procedures. More importantly, before estimating any nonlinear model one has to test if the series under investigation is linear or nonlinear in nature (Enders, 2014). The empirical literature reviewed is grouped into three sub-sections. The first outlines evidence on testing for linearity, the second on the estimation of nonlinear models and the third on nonlinear unit root tests.

There is considerable evidence in support of nonlinear adjustments in the RER in the literature. While there are several tests for nonlinearity, the most common is the Luukkonen-Saikkonen-Terasvirta (LST for short) by Luukkonen et al. (1988) and the Brock-Dechert-Scheinkman test (BDS for short) by Brock et al. (1996) test. The BDS test has a general nonlinear model alternative while the LST test has a specific smooth transition autoregressive alternative (ESTAR or LSTAR). Between the two, the BDS test has low power in small samples compared to the LST (Timo et al., 1992). Using the LST, literature has established that nonlinearities are quite common in Africa (Anoruo et al, 2006), Asia (Liew et al, 2003, 2004, 2006), Middle East (Sarno, 2000) and developed economies (Sarantis, 1999; Baum et al, 2001).

More importantly, the use of nonlinear models has overturned previously equivocal results on the mean reversion in the RER into relatively strong support for mean reversion across various settings including: seven advanced economies (Baum et al., 2001), 11 middle eastern economies (Sarno, 2000) [8], 11 Asian economies (Liew et al, 2003, 2006) and 13 African economies (Anoruo et al. 2006) [9]. Since most of the aforementioned studies exclusively use the LST, they resort to non-formal ways to select the best STAR model (ESTAR or LSTAR)[10]. Most studies have generally preferred the ESTAR because of its theoretical appeal in that rational economic agents are assumed to utilize arbitrage opportunities in a similar manner regardless of whether the RER is below or above the equilibrium value. However, asymmetric adjustment in RER cannot be ignored as Michael et al (19970 and Sarantis (1999) find evidence of that.

The study by Michael et al. (1997) was one of the earliest attempts to fit nonlinear models to RER dynamics[11]. They estimate five ESTAR and one LSTAR models to characterise RER deviations from PPP for six currency pairs of advanced economies. However, they use long span time series covering at least one century thereby mixing data from different regimes, which compromises the statistical inference of their results (Sarno, 2005). In addition, their study lacks nonlinear stationarity tests of the respective RERs as the nonlinear unit root tests

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[8] Sarno (2000) uses an error correction model linearity test by Granger & Terasvirta (1993) which is a compatriot of the univariate LST test.

[9] Zambia is not included in this sample.

[10] One common way is to eyeball the shape of the transition function for symmetry (ESTAR) or asymmetry (LSTAR)

[11] All studies in this paragraph already applied nonlinearity tests before estimating nonlinear models and that detail is omitted for brevity.

had not been developed by then. Taylor & Peel (2001) model PPP deviations as the nominal exchange rate deviations from equilibrium levels implied by standard monetary models. They find evidence in favour of ESTAR as opposed to LSTAR and using the formal Terasvirta (1994) test find that PPP deviations within 25% of equilibrium do not necessitate any adjustment in RERs. Taylor et al. (2001) model deviations of the RER from its mean for five advanced economies and estimate ESTAR models. The transition parameters were statistically significantly different from zero, implying nonlinear stationarity for all the countries. Baum et al. (2001) also estimate ESTAR models for 17 OECD countries and find that nonlinearity best captures RER deviations and that majority of the RER deviations from PPP, though persistent, were are not economically significant. From the above, it can be gleaned that most studies estimating nonlinear STAR models have been conducted in advanced economies with no known estimates from developing economies.

Testing for nonlinear mean reversion in the real exchange rate (PPP) rests on the use of a nonlinear unit root tests of the REER series. Nonlinear stationarity tests have more power than their linear stationary compatriots in small and large samples (Kapetanios et al., 2003). By far, the most widely used test for nonlinear stationarity is the Kapetanios-Shin-Snell (KSS) test (Kapetanios et al., 2003) who's null of a unit root is pitted against the alternative of a globally stationary ESTAR process. As this test was only developed in 2003, only studies after 2003 applied the KSS[12] (Liew et al. 2004; Liew et al. 2006; Anoruo, 2006; Zerihun & Breitenbach; 2016; Yildirim, 2017). In these studies, there is strong evidence in favour of PPP via nonlinear stationarity as opposed to the use of linear stationarity tests in earlier studies. Others like Arize (2011), Bahmani-Oskooee et al. (2008) also find evidence of nonlinear stationarity, but because they did not perform nonlinearity tests on the real exchange rate series, their results need to be taken with a pinch of salt. The study by Yildirim (2017) goes beyond the KSS to include the test developed by Kılıç (2011) and Kruse (2011) which are modifications or extensions of the KSS test. Kruse (2011) test extends the KSS to account for non-zero threshold values (i.e. mean reversion to non-zero values) while Kılıç (2011) is set up in a manner that implies that large currency appreciations or depreciations should revert to a zero equilibrium level as opposed to a "no arbitrage band". The conclusion from all three nonlinear stationarity tests favours the PPP for the Turkish Lira. Two notable exceptions to the use of KSS tests are Zerihun & Breitenbach (2016) and Cerrato & Sarantis (2006) which use the Fourier stationarity test and Sollis et al. (2002) tests, respectively[13]. Zerihun & Breitenbach (2016) find strong support for PPP for 11 Southern African Countries including Zambia. The Fourier stationarity tests however, do not provide additional information on the nonlinear dynamics as opposed to KSS, which is a test for nonlinear ESTAR type stationarity.

2. Model Specification, Methodology and Data Sources

This section first outlines the tests employed for nonlinearity, the nonlinear model estimated is described next and finally describes the nonlinear unit root tests employed.

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[12] Unless otherwise stated, all the studies included in this paragraph applied linearity tests before conducting nonlinear unit root tests

[13] The Sollis et al. (2002) tests are not commonly applied as they are a test for asymmetric nonlinear stationarity based on LSTAR model.

Testing for Nonlinearity and Selecting Appropriate Nonlinear Model

The Luukonen et al. (1988) or LST tests the null hypothesis that the series under investigation is linear and can be characterised by a linear autoregressive model whereas the alternative is that the series is a nonlinear smooth transition autoregressive process. As can be seen, rejection of the null hypotheses favors a nonlinear model (LSTAR or ESTAR) but the LST does not go further to select which of the two nonlinear models best fits the data. To choose the best nonlinear model, the Terasvirta (1994) and Escribano & Jorda (1999) tests are employed. The testing procedures for these two tests are done sequentially. They first test for nonlinearity and if evidence of nonlinearity is found, the contingent hypotheses to be tested is whether the series follows an ESTAR or LSTAR model. The test by Escribano & Jorda (1999) is more powerful than that of Terasvirta (1994) because Escribano & Jorda (1999) show that the Terasvirta (1994) test is likely to falsely detect LSTAR model in the presence of larger error variances or the presence of non-zero thresholds in a RER series (Escribano & Jorda,1999). Both tests are applied in this study and the ESTAR model is favoured.

Estimation of Nonlinear Model

The ESTAR model is estimated using nonlinear least squares. A general STAR model is usually presented as

$$q_t = \sum_{i=1}^{p} (\beta_i q_{t-i}) + \sum_{i=1}^{p} (\beta_i q_{t-i})\, \Phi[\theta : q_{t-d} - c] + \epsilon_t \tag{3}$$

where $\Phi[\theta : q_{t-d} - c]$ is the transition function that determines the degree of mean reversion and is governed by the parameter $\theta > 0$ which is the speed of mean reversion; the integer $d > 0$ is the delay parameter; and c is the threshold value. Since the ESTAR model is selected, the transition function $\Phi[\theta : q_{t-d} - c]$ in equation (3) will be the the exponential function shown in equation (4)

$$\Phi[\theta : q_{t-d}] = 1 - \exp\{-\gamma (q_{t-d} - c)^2\}, \quad \gamma > 0 \tag{4}$$

Substituting equation (4) into equation (3) yields the ESTAR model in which mean reversion behaviour is the same regardless of whether q_t is above or below the threshold value (c).

For purposes of this study's objectives, the most important parameters to be estimated will be the threshold value c and the speed of adjustment γ.

Nonlinear Unit Root Tests

Two nonlinear unit root tests are employed in this study, the KSS test by Kapetanios et al. (2003) and the Kruse (2011) test. Both tests are derived in a manner akin to the augmented Dickey Fuller test. Thus, the equation of interest is given in equation (5) as

$$\Delta\tilde{q}_t = \sum_{i=1}^{p} \rho_j \Delta\tilde{q}_{t-i} + \theta\tilde{q}_{t_{t-1}}\{1 - \exp(-\gamma(\tilde{q}_{t-1} - c)^2)\} + \varepsilon_t \tag{5}$$

where $\varepsilon_t \sim iid(0, \sigma^2)$; $\tilde{q}_t$ is the demeaned series of the real exchange rate i.e. $\tilde{q}_t = q_t - \mu_q$; c is the threshold value; and μ_q is the sample mean of q_t.

Testing for nonlinear stationarity is a test of whether the parameter for the speed of transition is statistically different from zero i.e.

$$\begin{aligned} &H_0: \gamma = 0 \quad \{\text{Not stationary}\} \\ &H_1: \gamma > 0 \quad \{\text{Nonlinear stationary}\} \end{aligned} \tag{6}$$

If the threshold value c in equation (5) is set equal to zero, then the hyotheses test in equation (6) is the KSS because it is designed to test for symmetric adjustment around a zero threshold value. Otherwise, the hypotheses tests in equation (6) form the Kruse (2011) test. Therefore, the Kruse (2011) test is essentially a modified version of the KSS by allowing the possibility that the real exchange rate may revert to a threshold value different from zero. Since γ is not identified under the null, testing for γ can only be possible through a Taylor series approximation of the ESTAR model to get an auxiliary regression that is used to derive LM-type test statistics[14]. As the asymptotic distribution of the KSS and Kruse (2011) statistics are not standard, critical values are obtained from Kapetanios et al. (2003) and Kruse (2011) respectively, as they are derived from Monte-Carlo simulations. Rejection of the null implies that the series is nonlinear stationary.

Data

Data was obtained from the Bank of Zambia for the period July 2000 to March 2020 at the monthly frequency giving a sample size of 223. This sample is chosen to reflect the shift in trading partners that occurred in 2000, which in turn changed the computation of the REER. This revised REER series what is currently being used by the Bank of Zambia.

3. Empirical Results and Analysis

It is most common to model deviations from PPP using a de-meaned series of the real effective exchange rate or real exchange rate. Therefore, a de-meaned real effective exchange rate series is analysed for the rest of this section and presented in the chart below.

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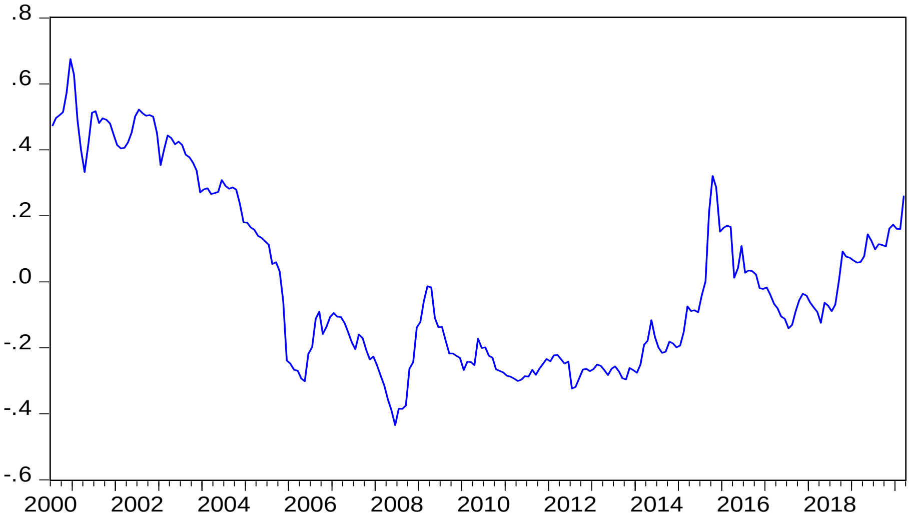
[14] See Kapetanios et al (2003) and Kruse (2011) for details on testing procedures

Time Plot of De-Meaned Kwacha Real Effective Exchange Rate



Source: Author's compilation from Bank of Zambia Data

The REER series declines steadily[15] from the beginning of the sample and bottoms out in 2008 when it begins to an upward trajectory until the end of the sample period. The REER exhibited smaller cyclical peaks and troughs, perhaps intimating some sort of oscillation of the REER between undervaluations and overvaluations. To demonstrate the poor performance of the linear unit root methodologies, a linear unit root test on the REER is conducted (Table 1).

Table 1: Linear Unit Root Rest on REER Series

	ADF^1	*ADF*^{2}	*PP*^{1}	*PP*^{2}
Actual value	-2.0030	-1.3960	-1.7958	-1.1093
Critical values				
1%	-3.458	-3.9976	-3.4580	-3.9974
5%	-2.8737	-3.4290	-2.8736	-3.4290
10%	-2.5740	-3.1380	-2.5733	-3.1379

Note: The superscript ^{1} denotes tests with intercept only while the superscript ^{2} denotes tests with intercept and trend. ADF is the augmented Dickey Fuller test and PP the Phillips Perron test

Linear unit root tests in Table 1 indicate that the REER is non-stationary at all conventional significance levels. This implies that PPP does not hold in the Zambian case hence the justification for non-linear models. The application of nonlinear models to the REER is done in a 3-step process: firstly, tests for nonlinearity and selection of appropriate nonlinear model are conducted. Secondly, the chosen nonlinear model is estimated. Finally, nonlinear unit root tests are conducted and interpreted.

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[15] A decline (rise) in the REER is effectively an appreciation (depreciation) since the nominal exchange in the REER computation is defined as Kwacha units per US Dollar.

Step 1: Testing for Nonlinearity and Selecting Appropriate Nonlinear Model

Application of the linearity test by Luukonen-Saikkonen and Teravirta (1988) reveals that the REER in Zambia is nonlinear in nature (Table 2). Thus, using a linear unit root testing procedure outlined above will likely bias the findings towards the rejection of the null due to low power. Evidence of nonlinear dynamics in the REER is also reported by Zerihun & Breitenbach (2016).

Table 2: Luukonen-Saikkonen-Terasvirta Tests for Linearity

Null Hypothesis	F-statistic	d.f.	p-value
$H_{04}: b_1 = b_2 = b_3 = b_4 = 0$	2.005031	(16, 187)	0.0147**
$H_{03}: b_1 = b_2 = b_3 = 0$	2.483545	(12, 191)	0.0048***
$H_{02}: b_1 = b_2 = 0$	1.222887	(8, 195)	0.2874
$H_{01}: b_1 = 0$	1.511764	(4, 199)	0.2001

Note: the b_i's are parameters from the LST auxiliary regression which if are all simultaneously null, confirms linearity otherwise, nonlinear smooth transition autoregressive model is appropriate; ** denotes significance at 5% level, *** denotes significance at 1% level, * denotes significance at 10% level

The LST test firmly establishes that REER is best modelled as a STAR model since H_{03} and H_{04} are rejected. To identify the appropriate STAR model to use, the Terasvirta (1994) and Escribano & Jorda (1999) tests are employed (Tables 3 and 4).

Table 3: Terasvirta (1994) Tests for Choosing Appropriate STAR Model

Null Hypothesis	F-statistic	d.f.	p-value
$H_{03}: b_3 = 0$	4.813536	(4, 191)	0.0010***
$H_{02}: b_2 = 0 \mid b_3 = 0$	0.935956	(4, 195)	0.4442
$H_{01}: b_1 = 0 \mid b_2 = b_3 = 0$	1.511764	(4, 199)	0.2001

Note: The Terasvirita (1994) rule is to choose an ESTAR if the p-value for H_{02} is smaller than H_{03}, otherwise the LSTAR model is selected; *** denotes significance at 1 % level

From Table 3, the Terasvirta (1994) test recommends the use of an LSTAR because the p-value for H_{03} is smaller than that for H_{02}. However, the more powerful Escribano-Jorda tests in Table 4 recommend the use of the ESTAR model as opposed to LSTAR[16].

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[16] The choice of ESTAR or LSTAR by Terasvirta (1994) and Escribano & Jorda (1999) can also be understood as a rejection of the linearity or REER

Table 4: Escribano-Jorda Tests for Choosing Appropriate STAR Model

Null Hypothesis	F-statistic	d.f.	p-value
$H_{0L} = b_2 = b_4 = 0$	2.3977	(8, 187)	0.0175**
$H_{0E} = b_1 = b_3 = 0$	2.1142	(8, 187)	0.0364**

Note: H_{0E} is the null that favours ESTAR if p-value of $H_{0E} >$ p-value of H_{0L} if p-values of H_{0E} and $H_{0L}$are < 0.05
** denotes significance at 5% level

All the three tests (LST, Terasvirta and Escribano-Jorda) have strictly rejected the use of a linear model in favour of a nonlinear STAR model. Further, the Terasvirta (1994) and Escribano-Jorda tests recommend the use of different LSTAR and ESTAR models, respectively. Estimating an LSTAR model implies that there is an asymmetric adjustment in the REER when it deviates from its equilibrium value, a plausible but not favoured perspective by economists as there are strong a priori expectations that REER adjustments should be the same for an under-valuation or over-valuation (Sarno, 2005). The ESTAR model is therefore selected based on the more powerful Escribano-Jorda test and parameter estimates are presented in the next section.

Step 2: Estimation of the ESTAR Model

$$\tilde{q}_t = 1.586\tilde{q}_{t-1} - 0.817\tilde{q}_{t-2} - 0.070\tilde{q}_{t-3} + 0.340\tilde{q}_{t-4} - [-0.513\tilde{q}_{t-1} +$$

$$0.6904\tilde{q}_{t-2}0.292\tilde{q}_{t-3} - 0.539\tilde{q}_{t-4}][1 - \exp\{-101.566(\tilde{q}_{t-5} + 0.098)^2\}]$$

(1.78) (29.7)

[0.0758] [0.000]

$R^2 = 0.966$; Breusch-Godfrey LM test =1.675 [0.1900]; DW = 1.981; S.E of Regression=0.040
t-statistics are in () and square brackets contain p-values and the serial correlation tests are part of robustness.

While only t-statistics and p-values for the slope and threshold parameters are indicated, the standard errors and p-values for the other coefficients are shown in the appendix, the majority of which are significant. The threshold variable is interpreted to mean that if the absolute value of the REER deviation from the mean is above 0.098, it shall take five months before REER begins to revert to equilibrium, all else being equal.

Other than checking that residuals are not serially correlated, the second most important diagnostic for nonlinear models is to look out for any remaining nonlinearity (Tables 5 and 6). The additive nonlinearity tests are performed on the residuals to check for any remaining additive nonlinear aspects that may not have been modelled. The tests rely on the Terasvirta (1994) and Escribano & Jorda (1999) tests which both strongly reject omission of additive nonlinearity.

Table 5: Additive Nonlinearity Tests

Terasvirta Tests

Null Hypothesis	F-statistic	d.f.	p-value
$H_{03}: b_3 = 0$	1.600268	(4, 185)	0.1760
$H_{02}: b_2 = 0 \| b_3 = 0$	1.434925	(4, 189)	0.2240
$H_{01}: b_1 = 0 \| b_2 = b_3 = 0$	2.191882	(4, 193)	0.0714*

Escribano-Jorda Tests

Null Hypothesis	F-statistic	d.f.	p-value
$H_{0L} = b_2 = b_4 = 0$	1.198408	(8, 181)	0.3022
$H_{0E} = b_1 = b_3 = 0$	1.388470	(8, 181)	0.2041

Note: For both Terasvirta and Escribano & Jorda tests, additional nonlinearity is confirmed if all hypotheses are rejected; * denotes significance at 10% level

The encapsulated nonlinearity test is a test for higher order regime STAR models. According to Table 6, there are no additional regimes beyond those identified by the nonlinear model estimated.

Table 6: Encapsulated Nonlinearity Tests

Terasvirta Sequential Tests

Null Hypothesis	F-statistic	d.f.	p-value
$H_{03}: b_3 = 0$	0.7326	(8, 175)	0.6625
$H_{02}: b_2 = 0 \| b_3 = 0$	1.6024	(7, 183)	0.1371
$H_{01}: b_1 = 0 \| b_2 = b_3 = 0$	1.4019	(7, 190)	0.2066

Escribano-Jorda Tests

Null Hypothesis	F-statistic	d.f.	p-value
$H_{0L} = b_2 = b_4 = 0$	1.1138	(15, 167)	0.3475
$H_{0E} = b_1 = b_3 = 0$	1.3618	(15, 167)	0.1716

Note: For both Terasvirta and Escribano & Jorda tests, additional nonlinearity is confirmed if all hypotheses are rejected.

Step 3: Nonlinear Unit Root Tests

Testing for PPP is essentially a test of stationarity i.e. whether the REER reverts to its long-run equilibrium value if deviations from the threshold values are large enough. Therefore, having estimated the nonlinear model, nonlinear unit root tests are performed on the REER.

From Table 7, both the KSS and the Kruse (2011) statistics are significant at the 5% level. This implies that the REER is stationary in a nonlinear manner thereby confirming that the PPP holds in Zambia.

Table 7: Nonlinear Stationarity Tests of KSS and Kruse (2011)

	KSS	KRUSE (2011)
STATISTIC	-3.20**	10.31**
CRITICAL VALUES		
1%	-3.48	13.75
5%	-2.93	10.17
10%	-2.66	8.60

Note *, ** and *** are the significance at 1%, 5% and 10% respectively

Linear unit root tests could not establish support for the existence of PPP during the floating exchange rate period. This perhaps signals the low power of linear unit root tests both in terms of short sample size and nonlinearities in the data. This is evidenced by the rejection of linear data generating processes in favour of the nonlinear process using the LST test. Therefore, the "no-mean reversion" conclusions by Pamu (2011) and Mokoena et al. (2008) may be due to their use of linear methodologies for the REER series whose data generating processes is nonlinear. Besides this study, only Zerihun & Breitenbach (2017) find evidence of nonlinear dynamics in the REER during the floating exchange rate period.

As regards the estimation of the nonlinear model, the most critical parameters i.e. threshold value and the speed of adjustment have been found to be significant. This shows that if the absolute deviation of REER from its mean exceeds 0.098, the REER is expected to begin its reversion to equilibrium after five months. It is a different research question altogether to establish which of the prices adjusts to equilibrium, whether it's the nominal exchange rate only, foreign price, domestic price, or a combination of the two or three of them. Using two nonlinear unit root tests to testing of PPP in the Zambian case, it has been found that both favour the assertion that the PPP holds in the Zambian case. The conclusions on the KSS echo those found by Arize (2011) and Bahmani-Oskooee et al. (2008) despite these studies using a sample period that spans fixed and floating exchange rate regimes and not testing for nonlinearity. To the best knowledge of the author, this is the first time the Kruse (2011) nonlinear unit root test is used in developing economies south of the Sahara, and for the Zambian economy. The only other known study by Yildirim (2016) also finds congruency in the conclusions between the KSS and Kruse (2011) tests for the Turkish economy. However, the Kruse (2011) test has more power as it allows for mean reversion to non-zero threshold values.

4. Conclusion

Linear models have been applied to the real exchange rates over the past 40 years, with equivocal results, casting doubt in the practical usefulness of the purchasing power parity theory. However, since most real exchange rates have nonlinear data generating processes, linear models have low power to detect evidence of mean reversion compared to their nonlinear counterparts. Zambia is no exception as most studies have shown no empirical support for mean reversion of the real exchange rate using linear methodologies. This study finds considerable evidence of nonlinear dynamics in the REER using a battery of linearity tests and, the exponential smooth transition autoregressive model is selected as the better nonlinear model to use. Nonlinear mean reversion of the REER is formally established using more powerful nonlinear unit root tests, implying that REER deviations larger than the estimated non-zero threshold values will not be persistent, but will eventually revert to equilibrium. Equilibrium in this case is not a point estimate but rather a range of values, (the "no-arbitrage band") whose upper limit is the estimated threshold value in absolute terms.

From a policy standpoint, unequivocal evidence of nonlinearity in the REER implies that not all REER deviations from PPP should be deemed as possible misalignments of the REER. Rather, the central bank should employ nonlinear models of the sort used in this study to provide some threshold values from which to assess the magnitude of REER deviations. Nonlinearity implies that the speed of adjustment to equilibrium is an increasing function of the REER deviation. Therefore, if REER deviations are judged to be large, monetary authorities, may decide to smoothen the anticipated sharp reversion in the REER directly (through measured intervention in the foreign exchange market) or indirectly through the adjusting policy rate. Additional areas of research worth considering would be to assess the half-life of shocks to REER deviations as well as identifying which variables are responsible for adjustment: nominal exchange rate, foreign prices, domestic prices or combination of all or some of these variables.

References

Abuaf N and Jorion P.1991. "Purchasing Power Parity in The Long Run." *Journal of Finance* 45,

Adler M and Lehman B. 1983. "Deviations from Purchasing Power Parity in the Long Run." *Journal of Finance* 39, 1471-1487.

Anoruo E, Liew VK and Elike U.2006. "Nonlinear Real Exchange Rate Behaviour." *International Research Journal of Finance and Economics Issue 1*

Arize A.2011. "Mean Reversion in Real Exchange Rates in Seventy LDCs: An empirical Investigation." *International Journal of Economics and Finance* Vol 3, No. 5

Bahmani-Oskooee M, Kutan AM and Zhou S. 2008. "Do Real Exchange Rates Follow a Nonlinear Mean Reverting Process in Developing Countries?" *Southern Economic Journal Vol 74. No.4*

Baum CF, Barkoulas JT and Caglayan M.2001. "Nonlinear Adjustment to Purchasing Power Parity in the Post Bretton Woods Era." *Journal of International Money and Finance* 20 379-399

Brock W, Davis D, Sheinkman José and LeBaron B .1996. "A Test for Independence Based on the Correlation Dimension." *Econometric Reviews. 15 (3), 197–235.*

Cerrato, M., Sarantis, N., 2006. "Nonlinear Mean Reversion in Real Exchange Rates: Evidence from Developing and Emerging Market Economies." *Economics Bulletin*. 7

Cheung Y and Lai K.1994. "Mean Reversion in Real Exchange Rates." *Economics letters* 46

Chipili J .2019. "Inflation Dynamics in Zambia." Forthcoming *African Economic Research Consortium Working Paper*

Diebold F, Husted S, and Rush M.1991. "Real Exchange Rates Under the Gold Standard." *Journal of Political Economy* 99, 1252-1271.

Dumas B.1992. "Dynamic Equilibrium and the Real Exchange Separated World." *Review of Financial Studies* 5 153-180

Chortareas G and Kapetanios G.2009. "Getting PPP Right: Identifying Mean-Reverting Real Exchange Rates in Panels." *Journal of Banking and Finance 33 390-404*

Enders W. 2014. "Applied Econometric Time Series." *Fourth Edition John Wiley and Sons inc.*

Escribano, A and Jordá O (1999). "Improved Testing and Specification of Smooth Transition Regression Models, in Nonlinear Time Series Analysis of Economic and Financial Data." Rothman, P., ed. ,Klewer: Boston. 289–319.

Frankel J. 1990. “Zen and the Art of Modern Macroeconomics: The Search for Perfect Nothingness.” In: Monetary Policy for a Volatile Global Economy. *Washington, DC: American Enterprise Institute.*

Frankel J and Rose A.1996. "A Panel Project on Purchasing Power Parity: Mean Reversion Within and Between Countries.” *Journal of International Economics Vol 40 (1)*

Frenkel JA.1978. "Purchasing Power Parity: Doctrinal Perspectives and Evidence from the 1920s.” *Journal of International Economics vol 8*

Froot K and Rogoff K .1994. "Perspectives on PPP and Long-Run Real Exchange Rates.” *NBER working paper series No. 4952*

Granger CWJ and Terasvirta T.1993. “Modelling Nonlinear Economic Relationships.” *Oxford University Press, Oxford*

Kanyembo C and Sheefeni JPS. 2013. “Purchasing Power Parity Between Zambia and South Africa.” *Journal of Emerging Issues in Economics, Finance and Banking* vol 2(5)

Kapetanios, G., Shin, Y., Snell, A., 2003. “Testing for a Unit Root in the Nonlinear STAR Framework.” *Journal of Econometrics vol 112*

Kılıç, R., 2011. “Testing for Unit Root in a Stationary ESTAR Process.” *Economic Review* 30 (3), 274-302.

Kruse, R., 2011. “A New Unit Root Test Against ESTAR Based on a Class of Modified Statistics.” *Statistics Papers* 52

Liew VK, Chong TT and Laim K .2003. “The Inadequacy of the Linear Autoregressive Model for Real Exchange Rates: Empirical Evidence from Asian Economies.” *Applied Economics* 35 1387-1392

Liew VK, Baharumshah AZ and Chong TT.2004. “Are Asian Real Exchange Rates Stationary?” *Economics letters* 83 313-316

Liew VK, Hock-ann L, Kian-Ping L and Huay-Huay L. 2006. “Linearity and stationarity of South Asian real exchange rates.” *MPRA no. 517 Available online* at https://mpra.ub.uni-muenchen.de/517/

Lothian JR and Taylor MP .1996. "Real Exchange Rate Behaviour: The Recent Float from the Perspective of the Past Two Centuries.” *Journal of International Money and Finance*, Vol. 17

MacDonald R .1996. “Panel Unit Root Tests and Real Exchange Rates”. *Economics Letters 50*, 7–11.

Mark N.1990. “Real Exchange Rates in the Long Run: An Empirical Investigation” *Journal of International Economics* 28, 115-136

Meese R. and Rogoff K .1988. "Was it Real? The Exchange Rate Interest Differential Relation Over the Modern Floating Exchange Rate Period." *Journal of Finance* 43, 933-948

Michael P, Nobay R, and Peel DA.1997. "Transaction Costs and Nonlinear Adjustments in Real Exchange Rates: An Empirical Investigation." *Journal of Political Economy 105*

Mokoena TM, Gupta, R and Van Eyden, R.2009. "Testing for PPP Using SADC Real Exchange Rates." *Southern African Journal of Economics Vol 77*

Moono H .2010. "An Empirical Test of Purchasing Power Parity Between Zambia and South Africa." *Master of Science in Economics* D*issertation, University of Sheffield*

Mungule OK .2004. "The Determinants of the Real Exchange Rate in Zambia." *African Economic Research Consortium Research paper 146*

Obstfeld M and Rogoff K.2000. "The Six Major Puzzles in International Macro-Economics: Is There a Common Cause?", *in NBER Macroeconomics Annual, ed. B. Bernanke and K. Rogoff (Cambridge, MA: MIT Press)*

O'Connell PGJ .1998. "The Overvaluation of Purchasing Power Parity". *Journal of International Economics* 44, 1–19.

O'Connell PGJ and Wei SJ.2002. "The Bigger They are, the Harder They Fall: Retail Price Differences Across US Cities". *Journal of International Economics* 55, 87-105.

Luukkonen R, Saikkonen P and Terasvirta T. 1988. "Testing Linearity Against Smooth Transition Autoregression". *Biometrika 72, 491-499.*

Pamu ME.2011. "Does the Copper Price Explain the Deviation of the Real Exchange Rate From Purchasing Power Parity Equilibrium in Zambia?"*The BOZ Reader:Issues on the Zambian Economy Vol 1. No. 6*, Bank of Zambia

Papell D. 2002. "The Great Appreciate, the Great Depreciation, and the Purchasing Power Parity Hypothesis." *Journal International Economics*, Vol. 57, No. 1.

Papell DH and Theodoridis H.2001. "The Choice of Numeraire Currency in Panel Tests of Purchasing Power Parity." *Journal of Money, Credit and Banking* 33, 790–803.

Rogoff K. 1996. "The Purchasing Power Parity Puzzle." *Journal of Economic Literature*, Vol. 34, No. 2.

Salvatore D. 2016. "International Economics." 12th Edition, Wiley and Sons.

Sarantis N. 1999. "Modelling Nonlinearities in Real Effective Exchange Rates." *Journal of International Money and Finance* 18 27-45

Sarno L. 2000. "Real Exchange Rate Behaviour in the Middle East: A Re-Examination." *Economics Letters 66 127-136*

Sarno L. 2005. "Towards a Solution to the Puzzles in Exchange Rate Economics: Where Do We Stand?" *The Canadian Journal of Economics* Vol.38 No. 3

Sarno L and Taylor MP.1998. "The Real Exchange Rates Under the Recent Float: Unequivocal Evidence of Mean Reversion" *Economics Letters 60*

Sarno L and Taylor MP. 2002. "The Purchasing Power Parity and the Real Exchange Rate" *IMF Staff Papers Vol 49 No. 1*

Sercu P, Uppal R and Van Hulle C.1995. "The Exchange Rate in the Presence of Transactions Costs: Implications for Tests of Purchasing Power Parity."*Journal of Finance* 50, 1309-19

Sollis R, Leybourne S and Newbold P.2002. "Tests for Symmetric and Asymmetric Nonlinear Mean Reversion in Real Exchange Rates." *Journal of Money, Credit and Banking 34*

Taylor MA. 2002. "Century of Purchasing Power Parity." *Review of Economics and Statistics* 84

Taylor MP and Sarno L. 1998. "The Behaviour of Real Exchange Rates During the Post Bretton Woods Period." *Journal of International Economics* 46

Taylor MP and Peel D. 2000. "Nonlinear Adjustment, Long-Run Equilibrium and Exchange Rate Fundamentals." *Journal of international Money and Finance* 19 33-53

Taylor MP, Peel D and Sarno L. 2001. "Nonlinear Mean-Reversion in Real Exchange Rates: Toward a Solution to the Purchasing Power Parity Puzzles." *International Economic Review* vol. 42, No. 4

Terasvirta T .1994. "Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models." *Journal of the American Statistical Association* Vol. 89, No. 425 pp 208-218

Terasvirta T, TjØstheim D and Granger CWJ. 1992. "Aspects of Modeling Nonlinear Time Series." *Research Report 1992:1* University of Goteborg, Department of Statistics ISSN 0349-8034

Yildirim D.2017. "Empirical Investigation of Purchasing Power Parity for Turkey: Evidence from Nonlinear Unit Root Tests." *Central Bank Review Volume 17, Issue 2*

Zerihun MF and Breitenbach MC .2016. "Nonlinear Approaches in Testing PPP: Evidence from Southern African Development Community." *Economic Modelling* 56

APPENDIX

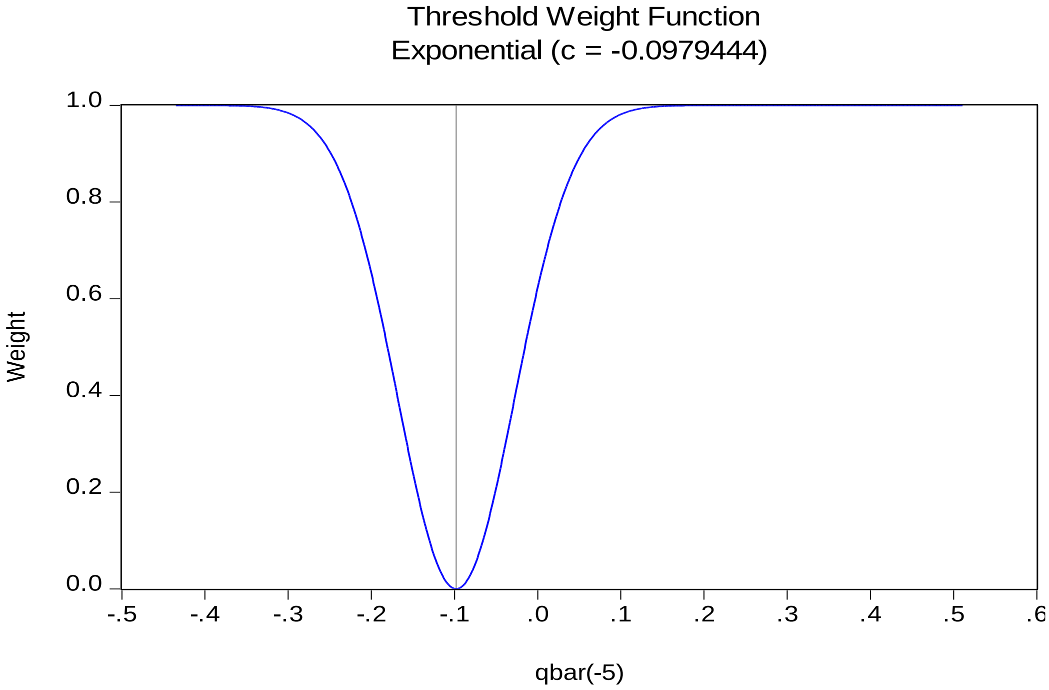
Threshold Autoregressive Model (De-meaned data)

Dependent variable q_t

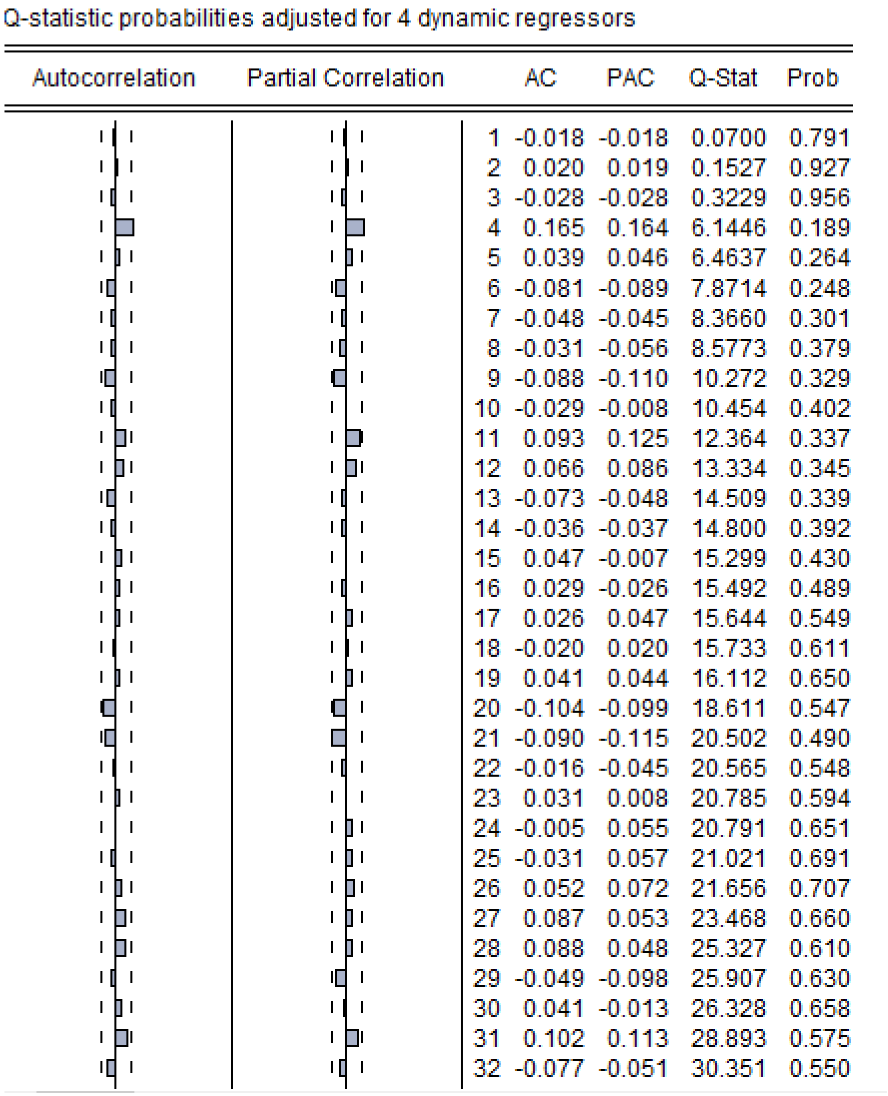
Variable	Coefficient	Std. Error	t-Statistic	Prob.
	Threshold Variables (linear part)			
$\tilde{q}_{t-1}$	1.586198	0.118077	13.43357	0.0000
$\tilde{q}_{t-2}$	-0.816553	0.173248	-4.713211	0.0000
$\tilde{q}_{t-3}$	-0.069916	0.211321	-0.330853	0.7411
$\tilde{q}_{t-3}$	0.339923	0.193620	1.755623	0.0807
	Threshold Variables (nonlinear part)			
$\tilde{q}_{t-1}$	-0.513147	0.162665	-3.154627	0.0019
$\tilde{q}_{t-2}$	0.689714	0.226010	3.051699	0.0026
$\tilde{q}_{t-3}$	0.291892	0.279319	1.045012	0.2973
$\tilde{q}_{t-3}$	-0.539357	0.205324	-2.626861	0.0093
	Slopes			
SLOPE	101.5644	56.89555	1.785103	0.0758
	Thresholds			
THRESHOLD	-0.097944	0.012591	-7.778974	0.0000

R-squared	0.965840	Mean dependent var	-0.070152
Adjusted R-squared	0.964280	S.D. dependent var	0.209873
S.E. of regression	0.039666	Akaike info criterion	-3.569565
Sum squared resid	0.309951	Schwarz criterion	-3.408564
Log likelihood	379.4500	Hannan-Quinn criter.	-3.504457
Durbin-Watson stat	1.981634		

Transition function Graph

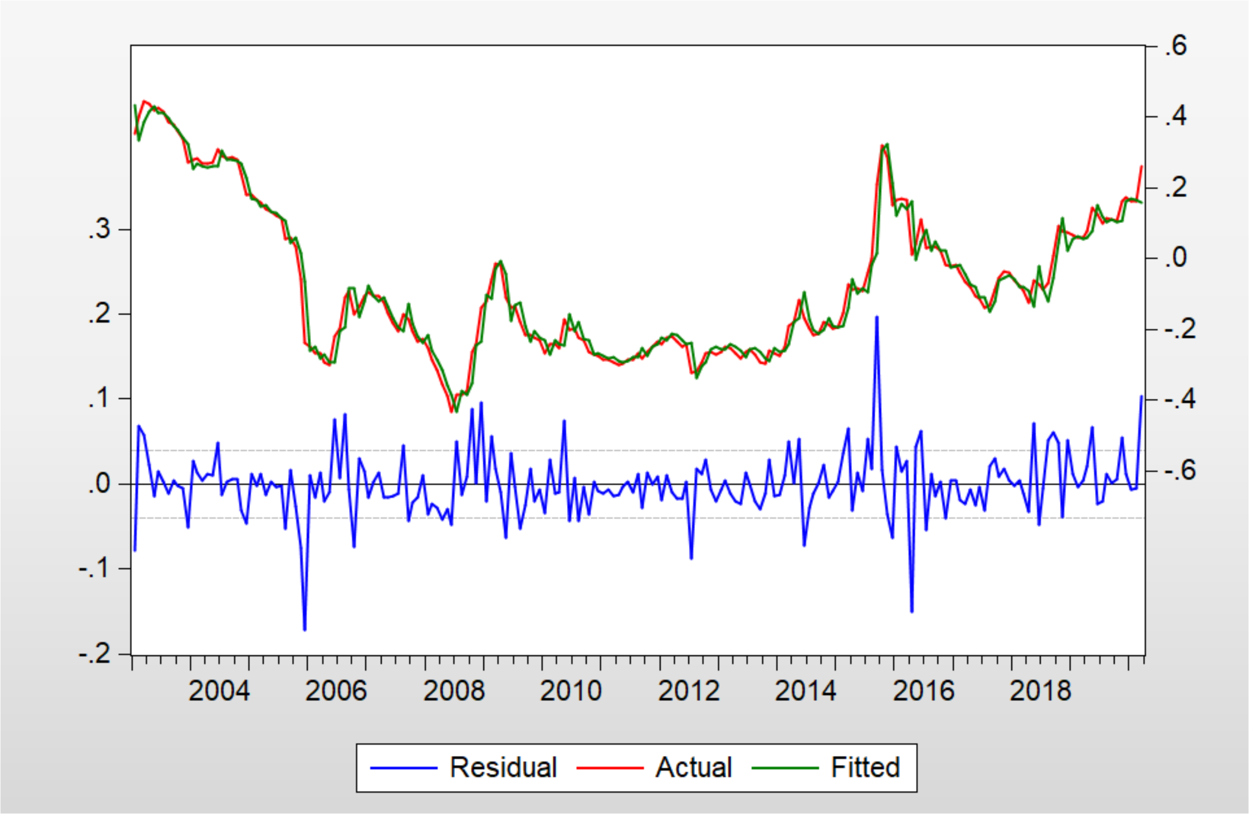


Correlogram for residuals from Threshold Regression

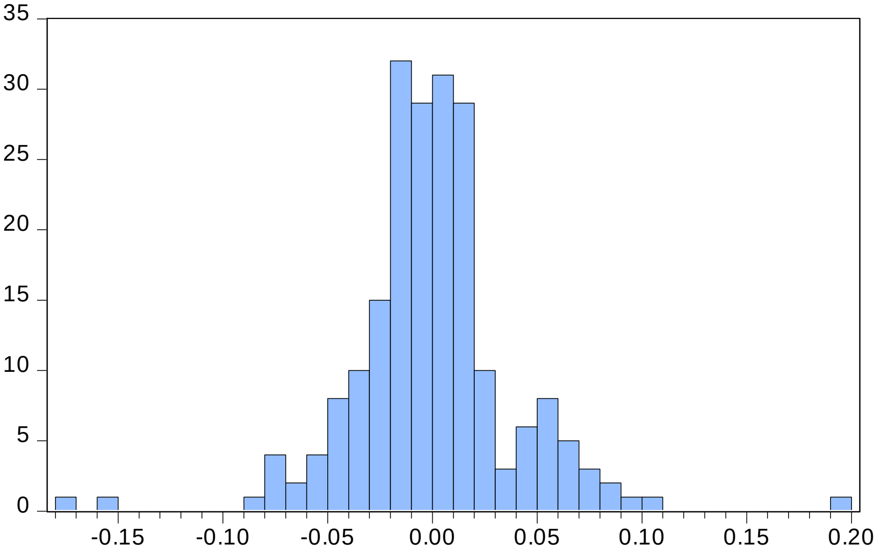


```
Q-statistic probabilities adjusted for 4 dynamic regressors

Autocorrelation   Partial Correlation      AC     PAC    Q-Stat   Prob

                                      1 -0.018  -0.018   0.0700  0.791
                                      2  0.020   0.019   0.1527  0.927
                                      3 -0.028  -0.028   0.3229  0.956
                                      4  0.165   0.164   6.1446  0.189
                                      5  0.039   0.046   6.4637  0.264
                                      6 -0.081  -0.089   7.8714  0.248
                                      7 -0.048  -0.045   8.3660  0.301
                                      8 -0.031  -0.056   8.5773  0.379
                                      9 -0.088  -0.110   10.272  0.329
                                     10 -0.029  -0.008   10.454  0.402
                                     11  0.093   0.125   12.364  0.337
                                     12  0.066   0.086   13.334  0.345
                                     13 -0.073  -0.048   14.509  0.339
                                     14 -0.036  -0.037   14.800  0.392
                                     15  0.047  -0.007   15.299  0.430
                                     16  0.029  -0.026   15.492  0.489
                                     17  0.026   0.047   15.644  0.549
                                     18 -0.020   0.020   15.733  0.611
                                     19  0.041   0.044   16.112  0.650
                                     20 -0.104  -0.099   18.611  0.547
                                     21 -0.090  -0.115   20.502  0.490
                                     22 -0.016  -0.045   20.565  0.548
                                     23  0.031   0.008   20.785  0.594
                                     24 -0.005   0.055   20.791  0.651
                                     25 -0.031   0.057   21.021  0.691
                                     26  0.052   0.072   21.656  0.707
                                     27  0.087   0.053   23.468  0.660
                                     28  0.088   0.048   25.327  0.610
                                     29 -0.049  -0.098   25.907  0.630
                                     30  0.041  -0.013   26.328  0.658
                                     31  0.102   0.113   28.893  0.575
                                     32 -0.077  -0.051   30.351  0.550
```

Residual vs fitted graph from Threshold Regression



Histogram of Residuals from Threshold Regression



Series: Residuals
Sample 2003M01 2020M03
Observations 207

Mean	-0.000184
Median	-0.002469
Maximum	0.197159
Minimum	-0.172036
Std. Dev.	0.038789
Skewness	0.210943
Kurtosis	8.195393
Jarque-Bera	234.3421
Probability	0.000000

The KSS test Regression Results

Dependent variable: $\Delta\tilde{q}_t$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$\Delta\tilde{q}_{t-1}$	0.311636	0.065246	4.776315	0.0000
$\Delta\tilde{q}_{t-2}$	-0.093651	0.068358	-1.370001	0.1721
$\Delta\tilde{q}_{t-3}$	-0.051669	0.068339	-0.756059	0.4504
$\Delta\tilde{q}_{t-4}$	0.116935	0.067966	1.720505	0.0867
$\Delta\tilde{q}_{t-5}$	-0.057222	0.065199	-0.877648	0.3811
$\tilde{q}_t^3$	-0.182038	0.056715	-3.209711	0.0015

R-squared	0.138003	Mean dependent var	-0.001793
Adjusted R-squared	0.118847	S.D. dependent var	0.042962
S.E. of regression	0.040329	Akaike info criterion	-3.557875
Sum squared resid	0.365941	Schwarz criterion	-3.468462
Log likelihood	416.9346	Hannan-Quinn criter.	-3.521812
Durbin-Watson stat	1.978074		

The Kruse (2011) test auxiliary regression

Dependent Variable: $\Delta\tilde{q}_t$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$\tilde{q}_t^3$	-0.246465	0.077642	-3.174398	0.0017
$\tilde{q}_t^2$	0.069854	0.037159	1.879870	0.0614
$\Delta\tilde{q}_{t-1}$	0.323959	0.065389	4.954317	0.0000
$\Delta\tilde{q}_{t-2}$	-0.081431	0.069388	-1.173566	0.2418
$\Delta\tilde{q}_{t-3}$	-0.032865	0.068919	-0.476863	0.6339
$\Delta\tilde{q}_{t-4}$	0.120977	0.066005	1.832839	0.0681

R-squared	0.140269	Mean dependent var	-0.001342
Adjusted R-squared	0.121248	S.D. dependent var	0.043417
S.E. of regression	0.040700	Akaike info criterion	-3.539662
Sum squared resid	0.374365	Schwarz criterion	-3.450522
Log likelihood	416.6007	Hannan-Quinn criteria	-3.503712
Durbin-Watson stat	1.977069		

Variance-Covariance Matrix for the Kruse (2011) Auxilliary Regression

	$\tilde{q}_t^3$	$\tilde{q}_t^2$	$\Delta\tilde{q}_{t-1}$	$\Delta\tilde{q}_{t-2}$	$\Delta\tilde{q}_{t-3}$	$\Delta\tilde{q}_{t-4}$
$\tilde{q}_t^3$	0.006028	-0.002022	-0.000187	-0.000882	-0.000577	-0.000668
$\tilde{q}_t^2$	-0.002022	0.001381	3.02E-05	0.000326	0.000261	0.000333
$\Delta\tilde{q}_{t-1}$	-0.000187	3.02E-05	0.004276	-0.001313	0.000547	0.000126
$\Delta\tilde{q}_{t-2}$	-0.000882	0.000326	-0.001313	0.004815	-0.001432	0.000633
$\Delta\tilde{q}_{t-3}$	-0.000577	0.000261	0.000547	-0.001432	0.004750	-0.001267
$\Delta\tilde{q}_{t-4}$	-0.000668	0.000333	0.000126	0.000633	-0.001267	0.004357

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